Divide both sides of Equation 9.25 by $((\boldsymbol{\Pi})_{[1][0]} - (\boldsymbol{\Pi})_{[0][0]})p(x|S = H_1)P(S = H_0)$,

$$\frac{p(x|S=H_0)}{p(x|S=H_1)} > \frac{(\boldsymbol{\Pi})_{[0][1]} - (\boldsymbol{\Pi})_{[1][1]}}{(\boldsymbol{\Pi})_{[1][0]} - (\boldsymbol{\Pi})_{[0][0]}} \; \frac{P(S=H_1)}{P(S=H_0)}[x] \tag{9.26}$$

Equation 9.26 should hold, *almost everywhere*, and presents a *lower bound* on the *likelihood ratio* for making the decision, $O = H_0$. The right hand side of Equation 9.26 does not depend on x. It is a constant which is only dependent on the associated penalties and the *a-priori* probability ratio of the two hypotheses H_0 and H_1.

Let us define the threshold,

$$\theta_{H_0} \stackrel{\Delta}{=} \log_b \left(\frac{(\boldsymbol{\Pi})_{[0][1]} - (\boldsymbol{\Pi})_{[1][1]}}{(\boldsymbol{\Pi})_{[1][0]} - (\boldsymbol{\Pi})_{[0][0]}} \; \frac{P(S=H_1)}{P(S=H_0)} \right) \tag{9.27}$$

as the *log-likelihood ratio threshold* for deciding in favor of the *null hypothesis*, H_0, against the *alternative hypothesis*, H_1. Deciding in favor of H_0, if

$$\log_b \frac{p(x|S=H_0)}{p(x|S=H_1)} > \theta_{H_0}[x] \tag{9.28}$$

and choosing H_1 otherwise, will give the *minimum error solution* or the, so called, *maximum a-posteriori solution* for a *binary hypothesis*. This result is also known as the *Neyman-Pearson lemma* [3]. As in before, any base may be picked for the computation of the logarithm – see Section 7.3.1.

Note that if we use the penalty matrix defined in Equation 9.9, then Equation 9.26 would be simplified as,

$$\frac{p(x|S=H_0)}{p(x|S=H_1)} > \frac{P(S=H_1)}{P(S=H_0)}[x] \tag{9.29}$$

or in other words,

$$\theta_{H_0} = \log_b \frac{P(S=H_1)}{P(S=H_0)} \tag{9.30}$$

9.2.2 Relative Information and Log Likelihood Ratio

Referring to Section 7.6, recall the expression for the *relative information* gained by observing $X = x$ in favor of hypothesis H_0 against H_1. As we saw, this *relative information* was given by Equation 7.78 as the difference between the logarithms