unit circle, to coincide with its Fourier transform version of the definition.[9]

The definition given by Equation 24.600, for $\tilde{h}_{pc}$, uses the square of the inverse z-transform. The square was used by [9] in order to be consistent with the original definition of Cepstrum by [4]. There are many different variations of the Cepstrum in the literature. most of them analyze the log of the power spectral density, but they differ in the final definition related to the spectral analysis of the log of the PSD. Equation 24.600 squares the inverse z-transform. At this point let us limit ourselves to the values on the unit circle in the z-plane, leading to a Fourier transform version of the power Cepstrum.

In the continuous-time, the power Cepstrum may be defined in terms of the inverse of the Fourier transform of the log of the PSD as follows,

$$\tilde{h}_{pc} = \left[\frac{1}{2\pi} \int_{-\pi}^{\pi} \log\left(|H(\omega)|^2 \right) e^{i\omega t} d\omega \right]^2 \tag{24.601}$$

Example 24.9 (Cepstrum of an Echo).
Consider the periodic signal of Equation 24.602 which is a combination of 4 *sinusoidal components,*

$$h(t) = 0.2\left(\sin(\omega t) + \sin(2\omega t) + \sin(3\omega t) + \sin(4\omega t)\right) \tag{24.602}$$

where $\omega = 2\pi f$ *and* $f = 80$ *Hz.*

Furthermore, consider a simple echo of this signal which arrives at a delay of $\tau = 0.3s$ *with a reflection factor of* $a = 0.4$ *governed by Equation 24.595. The new signal will include the original signal plus the echo. Equation 24.603 describes this new signal.*

$$x(t) = h(t) + 0.4h(t - 0.3) \tag{24.603}$$

Figure 24.22 shows the waveform and spectrogram representations of $x(t)$ of Equation 24.603. Figure 24.23 is a zoomed portion of the signal around the point in time when the echo arrives ($t = 0.3s$). In figure 24.24, we see the 4 peaks associated with the 4 different frequencies in the basic periodic signal. Finally, Figure 24.25 shows a plot of the power Cepstrum of the echo-contaminated signal of Equation 24.603. Note the peak at the arrival of the echo, $t = 0.3s$. The 1977 paper by Childers, et al. and its correction [9, 10] present a thorough treatment of the power Cepstrum, the complex Cepstrum and the phase Cepstrum. [35] also presents a good historical account of the Cepstrum for further reference.

[9] [4] simply used *Cepstrum* since it only defined the power Cepstrum. The *complex Cepstrum* was defined later by Oppenheim [36] in the process of developing the subject of *homomorphic system theory*.